IDENTIFICATION OF CRITICAL LINKS IN EMERGENCY RESCUE BASED ON THE GERT NETWORK

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ABSTRACT

When emergency happens on urban transportation network, there is usually not enough time for rescuers collecting traffic information and identifying critical links timely. Aimed at the problem, this article tries to propose a new method to solve it. Firstly, under poor information environment, we approximately estimate link’s average travel time by fuzzy mathematics theory, then establishes a GERT (Graph Evaluation and Review Technique, GERT) network model to reflect rescuer’s psychological preference and link selective behavior, and finally design a new algorithm to gain the shortest rescue path and finish identification of critical links in emergency. The paper selects an actual transportation network from a Chinese city to verify model and algorithm, the result shows that it is feasible to approximately estimate average travel time by fuzzy mathematics theory; compared with traditional method, the GERT network model can better reflect rescuer’s choice behavior in emergency and reduces previous method’s computational complexity from network level to path level, which is easily solved and has well effect.

KEYWORDS: emergency rescue; critical link; GERT network; poor information; fuzzy mathematics.

1 INTRODUCTION

Emergencies (like traffic accidents, natural disasters and festival parades) have the characteristics with complex factors, wide impact and high randomness on urban transportation network. Such accidents not only directly threaten the safety of people’s life and property, but also have a negative influence on network’s traffic state. Therefore, when there are emergencies in daily life (especially in rush hour), it is very important to precisely identify critical links between rescue origin location and destination location, which can help related personnel effectively control relevant links to prevent secondary congestion and find the best rescue path to arrive at destination location timely.

As it is understood, identification of critical links is the foundation of vulnerability analysis. The traditional research method is that constructing identification index firstly, then removing each link from transportation network in turn, and finally estimating the impact of deleted link on the
whole transportation network based on identification index’s variety. Currently, there have been two identification methods of critical links: direct measurement method and indirect measurement method.

The direct measurement method analyzes the relationship between link and traffic volume directly. For example, in order to identify the bottleneck of through capacity on transportation network, Jennelius et al. (1) firstly introduced the link importance degree to identify critical links. Based on Jennelius, Taylor et al. (2) used traffic assignment to gain traffic volume’s variety after removing each link from network and regarded the variation as link’s importance degree. Compared with existing studies, Scott (3) provided an important ideal that critical link should be the link that has great influence on transportation network, so identification of critical links should apply some network indexes to measure it. Like Scott, many scholars such as Berdica (4), Chen et al. (5) and Luathep et al. (6) also thought that it was necessary to establish more diversified indexes to indirectly evaluate the deleted links’ influence on network besides traffic volume. As a result, there have been new indirect indexes to replace traffic volume as link’s importance degree in recent years.

The indirect measurement method uses the variation of other traffic characteristics indexes to describe link’s importance on transportation network. For instance, on the basis of complicated network theory and reliability theory, D’Este et al. (7) and Hou (8) respectively adopted connectivity and congestion delay to depict link’s importance degree. Considering of network’s robustness under emergency circumstance, Sullivan et al. (9) established a new identification index called Network Robustness Index (NRI) to find the most critical link on network. Nevertheless, there are two weaknesses in such indirect measurement method. First, the method doesn’t guarantee the global optimality of the solution unless all possible situations are scanned. However, if there is an unexpected emergency on a large-scale transportation network, the number of possible situations can be prohibitively huge that it may be time-consuming to work out the outcomes. Second, due to using other indirect identification indexes which can’t truly reflect transportation network’s traffic state in rush hour, the possibility of misjudging critical links is greater than direct measurement method. Therefore, Wang et al. (10) and Farahani et al. (11) applied linearization techniques to develop a single-level mixed-integer linear programming (MILP) to approximately solve out the model built by indirect measurement method. Following Wang and Farahani, Zhang (12) improved the NRI by complicated network theory and adopted an optimization approach which is formulated as a transport network design problem (NDP) to reduce the possibility of misjudging critical links. Similarly, Li et al. (13) used some robustness index such as network efficiency and connectivity to deal with the NDP, which make it easier to find the most critical links.

It should be noted that, although the above researches have made useful exploration and positive contribution, identification of critical links in emergency rescue owns the following three significant differences:

1. Due to emergency’s urgency, it is difficult for rescuer to rapidly grasp enough information about transportation network in short time and related personnel must identify critical links under poor information environment.
2. Rescuers usually select rescue path on the basis of their travel experience and psychological preference. Hence traffic choice behavior should be taken into consideration in emergency rescue.
3. Different from common cars, rescue vehicles own a higher right of way (ROW) and are little affected by traffic flow distribution on transportation network.

In view of these differences, the adaptability of current methods is weak and it is necessary to make a further study on this problem. Therefore, this article is organized as follows: under poor information environment, we adopt fuzzy mathematics theory to approximately estimate rescue vehicles’ average travel time on link, then introduce the graph evaluation and review technique (GERT) to fuse average travel time with link selective probability, next, design a new calculation method of link importance degree to identify critical links. Finally, numerical examples are present to show how the proposed method is applied for emergency rescue.

2 APPROXIMATE ESTIMATION OF LINK’S AVERAGE TRAVEL TIME

Usually, link’s travel time is closely related with traffic condition, intersection delay and many other factors on urban transportation network. Due to rescue operation’s urgency, there isn’t enough
time for rescuer to collect much traffic information to make a foolproof rescue plan. So, it is very important to approximately estimate link’s average travel time in poor information environment.

2.1 Emergency rescue vehicles’ travel time on link

Vehicles’ travel time on link is usually divided into two parts: normal driving time and intersection delay. Considering that rescue vehicles own higher right of way (ROW) on transportation network, this article temporarily assumes that there is no intersection delay and only thinks about rescue vehicles’ driving time on link. Therefore, we can approximately calculate link’s travel time \( T_i \) as following:

\[
T_i = \frac{S_i}{v_i} \tag{1}
\]

Where \( S_i \) is link’s length; \( v_i \) is vehicles’ average travel speed on link.

For a particular transportation network, its basic information such as link’s length and road grade is predetermined. So we just need to figure out \( v_i \) and can directly obtain link’s travel time by the equation (1).

2.2 Fuzzy calculation of vehicles’ average travel speed on link

Considering each link’s basic information and its historical traffic condition, the article utilizes Fuzzy Mathematics Theory to approximately calculate vehicles’ average travel speed \( \overline{v_i} \), and its specific process is achieved by the following Algorithm I. (Note: the Algorithm I is mainly applicable for the situation that there is free traffic flow on transportation network and travelers all own high visibility in good weather.)

**Algorithm I**

Step 1 Defining factor discourse domain \( U \) and evaluation discourse domain \( V \).

The factor that affect rescue vehicles’ average travel speed constitutes factor discourse domain 

\[
U = \{U_1, U_2\} \tag{2}
\]

Where \( U_1 \) is road grade set (that owns 4 grades: urban expressway, backbone road, secondary main road and by-pass); \( U_2 \) is link’s congestion degree set (that is also called link’s saturation degree set which can be measured by \( V/C : V \) is link’s free traffic flow and \( C \) is link’s traffic capacity. Similarly, \( V/C \) usually owns 4 grades: <30%, 55%, 70% and >100% ).

The link’s design speed constitutes evaluation discourse domain \( V \):

\[
V = \{v_1, v_2, v_3, v_4\} \tag{3}
\]

Where \( v_1 \sim v_4 \) are respectively set as design speed values: 60km/h, 40km/h, 30km/h and 20km/h. (Note: the value standard is from Chinese Specification for Design of Municipal Roads (14).)

Step 2 Fuzzy relationship \( R \)’s building. Using evaluation discourse domain \( V \) respectively to evaluate road grade set \( U_1 \) and link’s congestion degree set \( U_2 \). And obtaining the corresponding fuzzy relationship \( R_i \) and \( R_2 \). The specific process is as following:

1. Building a correspondence between and like this: the design speed of road grade set \( U_1 \) (\( U_1 = \{urban\ expressway,\ backbone\ road,\ secondary\ main\ road,\ by-pass\}\) ) are respectively valued as 60km/h, 40km/h, 30km/h and 20km/h. So, we can gain a discrete fuzzy relation \( R_i \):

\[
R_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4}) \tag{4}
\]

Where \( r_{i1} \sim r_{i4} \) are relative importance that are individually set as 1, 1/2, 1/3 and 1/4 by the discrete fuzzy principle.

2.
Similarly, building another correspondence between $U_2$ and $V$ like this: the design speed of congestion degree set $U_2 = \{< 30\%, 50\%, 70\%, > 100\% \}$ are respectively valued as $60$km/h, $40$km/h, $30$km/h and $20$km/h. So, we can gain a continuous fuzzy relation $R_2$:

$$R_2 = \{r_{21}, r_{22}, r_{23}, r_{24}\}$$

Where $r_{21} \sim r_{24}$ are individually calculated as following:

$$r_{21} = \begin{cases} 
1, & x_i \leq d_1 \\
\frac{d_2 - x_i}{d_2 - d_1}, & d_1 < x_i < d_2 \\
0, & x_i \geq d_2 
\end{cases}$$

$$r_{22} = \begin{cases} 
\frac{d_2 - x_i}{d_2 - d_1}, & x_i < d_2 \\
\frac{x_i - d_1}{d_2 - d_1}, & d_1 \leq x_i \leq d_2 \\
0, & x_i \leq d_1, x_i \geq d_3 
\end{cases}$$

$$r_{23} = \begin{cases} 
\frac{d_3 - x_i}{d_3 - d_2}, & d_2 < x_i < d_3 \\
\frac{x_i - d_2}{d_3 - d_2}, & d_2 \leq x_i \leq d_3 \\
0, & x_i \leq d_3, x_i \geq d_4 
\end{cases}$$

$$r_{24} = \begin{cases} 
\frac{x_i - d_2}{d_4 - d_2}, & d_2 < x_i < d_3 \\
\frac{d_4 - x_i}{d_4 - d_2}, & d_2 \leq x_i \leq d_3 \\
1, & x_i \leq d_3, x_i \geq d_4 
\end{cases}$$

In the above equation (6), $d_1 \sim d_4$ indicate 4 link’s congestion degree grades :$30\%$, $55\%$, $70\%$ and $100\%$; $x_i$ indicates link $i$’s historical congestion degree.

Comprehensively considering $1$ and $2$, we can get a concrete form of fuzzy relation $R$ as following:

$$R = (R_1, R_2)^T$$

Step 3 Fuzzy relationship $R$’s adjustment. Obviously, factor set $U_1$ and $U_2$ have different influence on vehicles’ average travel speed. And compared with road grade set $U_1$, link’s congestion degree set $U_2$ has more important effect on vehicles’ average travel speed in daily life. So, individually giving $U_1$ and $U_2$ weight coefficients: $\alpha_1$ and $\alpha_2$ to adjust fuzzy relation $R$, and obtaining a new fuzzy relationship $R'$ as following:

$$R' = (\alpha_1, \alpha_2) \otimes R, \quad \alpha_1 < \alpha_2$$

Where $\otimes$ is the Zadeh fuzzy operator that is valued according to actual condition.

Step 4 Average travel speed $\overline{v}_i$’s calculation. Combining with fuzzy relation $R'$ and evaluation discourse domain $V$, we are able to get the final estimated values of vehicles’ average travel speed on link $i$:

$$\overline{v}_i = V \times (R')^T$$

Where evaluation discourse domain $V$ is link’s design speed grade set whose concrete form is $V = \{60, 40, 30, 20\}$.

Through the Algorithm I, this article obtains the estimated value of average travel speed $\overline{v}_i$ on link $i$ in poor information environment, and then we can calculate travel time $T_i$ by the equation (1).

Considering that average travel time is a major influence factor for rescuer’s link selective behavior, therefore, we will combine link selective probability $p(T_i)$ with average travel time $T_i$ to carry on the GERT network analysis in the next stage.
3 THE LINK SELECTIVE PROBABILITY BASED ON THE GERT NETWORK

Mixing probability theory and optimization theory, the Graph Evaluation and Review Technique (GERT) firstly appeared in the critical path method which is aimed at solving network planning problem.

This paper regards urban transportation network as a random network with link selective probability, then defines link’s moment generating function to quantify each link’s average travel time $T_i$ and selective probability $p(T_i)$, finally designs a new algorithm to find the shortest rescue path on the GERT network.

3.1 The basic mathematical properties of moment generating function

**Definition:** for random variable $X$ and any real number $s$, setting $M_X(s)$ as the moment generating function of random variable $X$, the concrete form of $M_X(s)$ is shown as following:

$$M_X(s) = E(e^{sx}) \begin{cases} \int_{-\infty}^{+\infty} e^{sx} f(x) dx, & \text{when } X \text{ is a continuous random variable} \\ \sum_{x} e^{sx} p(x), & \text{when } X \text{ is a discrete random variable} \end{cases}$$

(10)

Where $f(x)$ is probability density function; $p(x)$ is probability distribution. $M_X(s)$ has two important properties that will be widely used on the GERT random network.

**Property 1:** If random variable $X$ is bounded, mathematical expectation $E(e^{sx})$ in the equation (10) is existent for any real number $s$.

**Property 2:** Taking the derivative of moment generating function $M_X(s)$ by $s$. When $s = 0$, the derivative’s value is the origin moment of random variable $X$. The concrete deduction process is shown as following:

$$\left[ \frac{\partial}{\partial s} M_X(s) \right]_{s=0} = \left[ \frac{\partial}{\partial s} \int_{-\infty}^{+\infty} e^{sx} f(x) dx \right]_{s=0}$$

$$= \left[ \int_{-\infty}^{+\infty} x e^{sx} f(x) dx \right]_{s=0}$$

$$= \int_{-\infty}^{+\infty} x f(x) dx = E[X]$$

(11)

The above equation (11) can also be written as $M_X'(0) = E[X]$. Similarly, if $X$ is a discrete random variable, we can get the same calculating result with the equation (10). So there is no more detail.

3.2 The GERT network’s analysis on transportation network

Setting an actual transportation network as $G = (N, I)$ where intersections constitute a node set $N$ and links constitutes a edge set $I$. Random variable $T_i$ indicates the average travel time on link $i$ ($i \in I$) which connecting node $j$ and node $k$ on the network $G$, $j, k \in N$.

According to nodes’ logical sequence from origin location to destination location, average travel time $T_i$ is existent only in the case that rescue vehicles arrive at node $j$ and will select link $i$ to reach node $k$. So it is necessary to know the probability whether link $i$ will be selected when rescue vehicles arrive at node $j$. That is to say, we should know the probability distribution $p_{jk}(T_i)$ of random variable $T_i$.

Adopting the traveller’s experiential choice equation to calculate $p_{jk}(T_i)$, which is shown as following:
\[ p_{jk}(T_i) = \frac{\exp(\beta - \gamma T_i)}{\sum_{i=1}^{m} \exp(\beta - \gamma T_i)}, \quad i = 1, 2, 3, ..., m \in I \]  

Where \( \beta \) and \( \gamma \) are the parameters that can be fit by disaggregate method. Through the preceding work, average travel time \( T_i \) has been approximately estimated and its result constitutes a discrete bounded set, so we can use the Definition to depict each link’ s moment generating function \( M_{jk}(s) \):

\[ M_{jk}(s) = \sum_{for \ all \ T_i} e^{\gamma T_i} p_{jk}(T_i) \]  

Where \( p_{jk}(T_i) \) is the probability whether link \( i \) is selected when rescue vehicles arrive at node \( j \).

Further, setting \( W_{jk}(s) \) as link \( i \)'s transfer function, and let the following equation workable:

\[ W_{jk}(s) = p_{jk}(T_i) \cdot M_{jk}(s) \]  

After the above parameters conversion, we can just use a transfer function \( W_{jk}(s) \) to replace travel time \( T_i \) and moment generating function \( M_{jk}(s) \). The concrete form is shown as following:

![Figure 1: Parameter conversion on link i](image)

From the above figure, the transfer function \( W_{jk}(s) \) fuses travel time \( T_i \) with link selective probability \( p_{jk}(T_i) \) successfully. And according to the Property 1, \( W_{jk}(s) \)'s mathematical expectations \( E\left(e^{\gamma T_i}\right) \) is existent and it can be regarded as link \( i \)'s generalized impedance in the paper.

Now considering that a complete emergency rescue path is an ordered connection with series structure, so its geometric topological form is presented as a GERT trip chain which is shown as following:

![Figure 2: The trip chain of GERT network with series structure](image)

From the above figure, we know that the probability whether rescue vehicles arrive at node \( k \) is only related with link \( i \)'s selective probability \( p_{jk}(T_i) \) and its transfer function \( W_{jk}(s) \), and it is an “one step transmit process” from node \( k \) to the next node \( l \) (that is to say rescue vehicles will arrive at node \( l \) without passing any other intermediate nodes ). Therefore, for the series structure \( j \rightarrow k \rightarrow l \) on the above figure, its generalized total impedance in node \( k \) can be expressed as:

\[ E(T) = p_{jk}(T_i)T_i + W_{kl}(s)T_{i+1} \]  

Through further analysis, if rescue vehicles start from node 1 (rescue origin location) and will go through node 2, node 3, ..., node \( n-1 \) to reach node \( n \) (rescue destination location), the generalized total impedance of the GERT trip chain can be extended from the equation (15):
According to the Property 2, when \( s = 0 \),
\[
W_E(0) = p_E \cdot M_E(0) = \sum_{\forall t} e^t \cdot p_{jk}(t)|_{t=0} = p_E
\]  

It is shown that: the link’s selective probability \( p_E \) is equal to the transfer function \( W_E(s) \): \( p_E = W_E(s)|_{s=0} \). Using this mathematical property and combining with the equation (16), we can get a new form of the GERT trip chain’s generalized total impedance:
\[
E^*(T)|_{s=0} = W_{12}(s)T_1 + W_{23}(s)T_2 + W_{34}(s)T_3 + \ldots + W_{n-1n}(s)T_n|_{s=0}
\]  

Applying recursion method to deal with the equation (18), and we can easily getting every link’s generalized impedance on the GERT trip chain: \( E(T_i) = W_i \cdot T_i, \ i \leq n-1 \).

Note: (1) generalized impedance \( E(T_i) \) doesn’t indicate the real average travel time that rescue vehicles spend on link \( i \). In fact, when rescue vehicles select link \( i \), the real travel time is still \( T_i \). But \( E(T_i) \) considers of link selective probability \( p_{jk}(T_i) \) of rescue vehicles when they meet intersections on transportation network; (2) by analyzing the \( E(T_i) \)’s calculation form: \( W_i \cdot T_i \), the paper finds that the link with shorter travel time \( T_i \) usually has a bigger transfer function \( W_i \), which will make its generalized impedance \( E(T_i) \) is larger than the link that has longer travel time \( T_i \) but smaller transfer functions \( W_i \).

Therefore, although spending longer travel time, there is still a possible situation that rescuer would choose links with smaller \( E(T_i) \) instead of ones with bigger \( E(T_i) \). This phenomenon is especially evident in rush hour, because at that time links with shorter average travel time are usually more crowded and it will increase the possibility that rescuer has to select other links with lower congestion degree to reach rescue destination location as soon as possible.

### 3.3 Identification of critical links

Under the condition with known rescue origin/destination location, this article firstly removes link \( i \) from transportation network \( G(N, I) \), then regards generalized impedance \( E(T_i) \) as link’s weight and utilizes the shortest path algorithm (the Dijkstra algorithm) to respectively find out the corresponding shortest paths: \( L_G \) and \( L_{G-i} \cdot L_G \) is the initial shortest path on the transportation network \( G \) and \( L_{G-i} \) is the new shortest path on the \( G \) without link \( i \).

Then, individually calculating \( L_G \) ’s and \( L_{G-i} \)’s generalized total impedance \( E_G^*(T) \) and \( E_{G-i}^*(T) \) by the equation (18), and using their difference absolute value as link \( i \)’s importance degree:
\[
\Delta E_i^*(T) = |E_G^*(T) - E_{G-i}^*(T)|
\]  

The specific process is achieved by the following Algorithm II:

**Algorithm II**

**Step1.** Choosing the initial network \( G(N, I) \). \( I \) is link set and link’s total number is \( m \); \( N \) is node set and node’s total number is \( n \);

**Step2.** Using the Algorithm I to approximately estimate link’s average travel speed \( \overline{v}_i \) and obtaining link’s travel time \( T_i \) by the equation (1), the result is a set denoted by \( T = \{ T_i | i \leq m \} \).
Step3. Using the equation (12) to calculate each link’s experiential selective probability \( p_i \), the result is denoted by \( p = \{ p_i \mid i \leq m \} \). Putting set \( T \) and set \( P \) together to constitute a new set denoted by \( R = \{ (T_i, p_i) \mid i \leq m \} \). And having \( R \) load into the \( G(N, I) \) to generate the GERT network \( G^* (N, I, R) \);

Step4. Based on set \( R \), separately calculating link \( i \)’s moment generating function \( M_i \) and transfer function \( W_i \) by the equation (13) and equation (14). And getting link \( i \)’s generalized impedance denoted by \( E(T) = \{ E(T_i) = W_i \cdot T_i \mid i \leq m \} \)

Step5. On the GERT network \( G^*(N, I, R) \), utilizing the Dijkstra algorithm to obtain the shortest path \( L_G \) between rescue origin location and destination location. \( L_G \) is a ordered vector composed of \( s \) links, which is denoted by \( L_G = (l_1, l_2, ..., l_s) \subseteq G^* \). Then calculating the shortest path \( L_G \)’s generalized total impedance \( E_G^*(T) \) by the equation (18);

Step6. Deleting link \((1 \leq i \leq s)\) from \( G^* \) and updating the GERT network \( G^*(N, I, R) \). Then using the Dijkstra algorithm again to get the new shortest path \( L_{G-i} \) and corresponding generalized total impedance \( E_{G-i}^*(T) \) between rescue origin location and destination location.

Step7. Calculating link \( i \)’s importance degree by the equation (19). If \( i > s \), ending the algorithm; else, \( i = i + 1 \) and return to Step6.

4 CASE ANALYSIS AND CRITICAL LINK’S IDENTIFICATION

This article selects the transportation network \( G(N, L) \) constituted of 14 nodes and 21 links from Chengdu City as case analysis. And its concrete topological morphology is shown in the figure 3; each link’s congestion degree data from historical statistics is shown in the table 1:

![Example network](image)

The rescue origin location: 1
The rescue destination location: 5, 10, 14

Figure 3: Example network

<table>
<thead>
<tr>
<th>Link ( i ) number</th>
<th>Link ( i ) Congestion degree ( x_i )</th>
<th>Link ( i ) number</th>
<th>Link ( i ) Congestion degree ( x_i )</th>
<th>Link ( i ) number</th>
<th>Link ( i ) Congestion degree ( x_i )</th>
</tr>
</thead>
<tbody>
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<td>14</td>
<td>0.40</td>
<td>21</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 1 Historical congestion degree of links on the \( G(N, L) \)
Let the node 1 be the rescue origin location. And considering of the situation that there will be different rescue distances between rescue origin location and destination location, we select 3 different nodes as rescue destination locations: the node 5, the node 10 and the node 14. (The node 5 stands for the destination with short distance; the node 10 stands for the destination with medium distance; and the node 14 stands for the destination with long distance.)

4.1 Estimating link’ average travel speed

Estimating average travel speed by the Algorithm 1. In the Algorithm 1’s Step 3, we need to give different weight coefficients to different influence factors: $\alpha_1 = 0.15$ and $\alpha_2 = 0.85$, and then select a suitable Zadeh fuzzy operator $\otimes$ to approximately calculate average travel speed $\bar{v}_i$.

From fuzzy mathematics theory, we know that there are 2 Zadeh fuzzy operators whose estimating results are significantly different from each other: $(\bullet, +)$ and $(\wedge, \lor)$. The operator $(\bullet, +)$’s calculation form is $R'_j = \sum_{i}^{4} \alpha_i \cdot r_{ij}$. On the other hand, the operator $(\wedge, \lor)$’s calculation form is $R'_j = \max_i \min \left[ \alpha_i, r_{ij} \right]$. The paper based on these two operators uses the *matlab* 2012a to code the Algorithm 1 and respectively records computing results as $v_1$ and $v_2$, which is shown as the following figure (4).

![Figure 4: Two estimating results of link’ average travel speed](image)

From the above figure, the mean absolute error of these 2 operators is tiny:

$$\Delta v = \frac{\sum_{i=1}^{21} |v_1(i) - v_2(i)|}{21} = 2.744km / h$$

The result reflects that it is low-error to approximately estimate average travel speed $\bar{v}_i$ by fuzzy mathematics theory. In addition, the figure 4 also shows that there are 95 percent of average travel speed values (20 sample data) distributing in the speed interval [30km/h, 60km/h], which is accordant with the fact that rescues vehicles usually are driven at this speed on urban transportation network in daily life.

This article chooses the values estimated by the $(\bullet, +)$ operator (as shown on the table 2) to finish the next study.
Table 2 The average travel speed values estimated by the \((\ast, +)\) operator

<table>
<thead>
<tr>
<th>Link number</th>
<th>Travel speed (v_i/(\text{km/h}))</th>
<th>Link number</th>
<th>Travel speed (v_i/(\text{km/h}))</th>
<th>Link number</th>
<th>Travel speed (v_i/(\text{km/h}))</th>
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<tr>
<td>4</td>
<td>46.5500</td>
<td>11</td>
<td>36.6333</td>
<td>18</td>
<td>46.5500</td>
</tr>
<tr>
<td>5</td>
<td>37.4833</td>
<td>12</td>
<td>36.3500</td>
<td>19</td>
<td>40.3167</td>
</tr>
<tr>
<td>6</td>
<td>14.2500</td>
<td>13</td>
<td>38.9000</td>
<td>20</td>
<td>55.0500</td>
</tr>
<tr>
<td>7</td>
<td>50.2900</td>
<td>14</td>
<td>58.4500</td>
<td>21</td>
<td>56.4100</td>
</tr>
</tbody>
</table>

Based on the table 2, we use link’s average travel speed \(v_i\) to calculate link’s travel time \(T_i\) and measure link’s selective probability \(p_{jk}(T_i)\) with \(T_i\), which is necessary for us to establish the GERT network.

4.2 Calculating link’s generalized impedance

After working out \(T_i\) with the equation (1), we can generate the GERT network \(G^* = (N, I, R)\) and calculate each link’s generalized impedance \(E(T_i)\) by Step 3~Step 4 of the Algorithm II. The calculating result is shown as following:

Table 3 Each link’s travel time and generalized impedance

<table>
<thead>
<tr>
<th>Number (i)</th>
<th>Length (S_i/\text{km})</th>
<th>Travel time (T_i/\text{min})</th>
<th>Generalized impedance (E(T_i)/\text{min})</th>
<th>Number (i)</th>
<th>Length (S_i/\text{km})</th>
<th>Travel time (T_i/\text{min})</th>
<th>Generalized impedance (E(T_i)/\text{min})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>1.3582</td>
<td>1.6032</td>
<td>12</td>
<td>0.5</td>
<td>0.8253</td>
<td>2.2556</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>2.1810</td>
<td>1.5222</td>
<td>13</td>
<td>0.5</td>
<td>0.7712</td>
<td>2.9298</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.9417</td>
<td>1.0929</td>
<td>14</td>
<td>0.5</td>
<td>0.5133</td>
<td>1.7678</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
<td>1.0311</td>
<td>2.2351</td>
<td>15</td>
<td>1.0</td>
<td>1.3378</td>
<td>1.8854</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>1.2806</td>
<td>2.2175</td>
<td>16</td>
<td>0.8</td>
<td>1.3205</td>
<td>2.1758</td>
</tr>
<tr>
<td>6</td>
<td>2.3</td>
<td>3.6842</td>
<td>2.1878</td>
<td>17</td>
<td>1.2</td>
<td>1.2919</td>
<td>2.6252</td>
</tr>
<tr>
<td>7</td>
<td>1.2</td>
<td>1.4317</td>
<td>1.3203</td>
<td>18</td>
<td>0.5</td>
<td>0.6445</td>
<td>2.3268</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>1.5315</td>
<td>1.4804</td>
<td>19</td>
<td>0.5</td>
<td>0.7441</td>
<td>2.4168</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>1.1906</td>
<td>1.9587</td>
<td>20</td>
<td>1.0</td>
<td>1.0899</td>
<td>2.0771</td>
</tr>
<tr>
<td>10</td>
<td>1.2</td>
<td>1.2465</td>
<td>2.8674</td>
<td>21</td>
<td>2.1</td>
<td>2.2336</td>
<td>2.1390</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
<td>0.8189</td>
<td>1.5938</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the above table 3, we know that: ① although the links have the same length, their average travel time \(T_i\) are different (such as link 3, 4, 5 or link 11, 12, 13, 14). It shows that besides length \(S_i\), link’s average travel time \(T_i\) is also closely related with its average travel speed \(v_i\) and congestion degree \(x_i\) which have great influence on emergency rescuer’s link selective probability \(p_{jk}(T_i)\); ② the values are obviously different between average travel time \(T_i\) and generalized impedance \(E(T_i)\). For example, link 8’s travel time (1.5313 min) is bigger than link 5’s (1.1906 min), however, link 8’s generalized impedance (1.4804 min) is smaller than link 5’s (1.9587 min). It is interesting and reflects the fact that evaluating a link’s impedance should consider link’s selective probability \(p_{jk}(T_i)\) and it is more reasonable to regard link’s generalized impedance \(E(T_i)\) as the shortest path’s calculating standard instead of link’s average travel time \(T_i\).

Therefore, not only link’s length \(S_i\) but also its average travel time \(T_i\) and congestion degree \(x_i\) can obviously affect its generalized impedance \(E(T_i)\).
4.3 Measuring links’ importance degree

From the table 2, we adopt the matlab 2012a to code step 5~step 7 of the Algorithm II and look for the shortest path on the GERT network \( G^* = (N, I, R) \). There are 3 different shortest rescue paths shown as following:

![Diagram of emergency rescue paths]

Figure 5: The emergency rescue path with 3 different distances

From the above figure 5, we can get every situation’s rescue origin/destination location and its shortest path. And now let’s respectively expound them as following:

① the short rescue distance: from the node 1 to the node 5
When emergency occurs in the node 5, the shortest emergency rescue path is:

\[ 1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5 \]

Its generalized total impedance is \( E_{g^*}(T) = 2.4132 \text{ min} \). For each link on this path (link 3 and link 7), the paper obtains each link’s importance degree as following table 4:

<table>
<thead>
<tr>
<th>Delete link Number</th>
<th>New shortest path ( L_{u_{i-1}} )</th>
<th>New generalized total impedance ( E_{g^*}(T) )</th>
<th>Importance degree ( \Delta E^*(T)/\text{min} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1 ( \rightarrow ) 2 ( \rightarrow ) 4 ( \rightarrow ) 7 ( \rightarrow ) 5</td>
<td>3.8383</td>
<td>1.4251</td>
</tr>
<tr>
<td>7</td>
<td>1 ( \rightarrow ) 2 ( \rightarrow ) 4 ( \rightarrow ) 7 ( \rightarrow ) 5</td>
<td>3.8383</td>
<td>1.4251</td>
</tr>
</tbody>
</table>
In the table 4, it is unexpected to find that although link 3’s and link 7’s generalized
impedances $E(T_i)$ are different in the table 2, their importance degree $\Delta E^*_i(T)$ in the GERT
network $G^*=(N,I,R)$ both are 1.4251 min. Therefore, for the above short rescue
distance: $1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5$, link 3 and link 7 are equally critical for emergency rescuer. This
phenomenon reflects link’s importance degree $\Delta E^*_i(T)$ is related with the shortest rescue path $L_G$.

② the medium rescue distance: from the node 1 to the node 10
When emergency occurs in the node 10, the emergency rescue path is:
$1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 11 \rightarrow 9 \rightarrow 15 \rightarrow 10$

Its generalized total impedance $E^*_G(T) = 5.8924 \text{ min}$. For each link on this path (link 1, link 4, link 11 and link 15), the paper similarly obtains their importance degree $\Delta E^*_i(T)$ as following table 5:

<table>
<thead>
<tr>
<th>Delete link Number $i$</th>
<th>New shortest path $L_{G,i}$</th>
<th>New generalized total impedance $E^*_{G,i}(T)/\text{min}$</th>
<th>Importance degree $\Delta E^*_i(T)/\text{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 11 \rightarrow 9 \rightarrow 15 \rightarrow 10$</td>
<td>7.3175</td>
<td>1.4251</td>
</tr>
<tr>
<td>7</td>
<td>$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 11 \rightarrow 9 \rightarrow 15 \rightarrow 10$</td>
<td>7.3175</td>
<td>1.4251</td>
</tr>
<tr>
<td>11</td>
<td>$1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 8 \rightarrow 6 \rightarrow 12 \rightarrow 10$</td>
<td>6.1492</td>
<td>0.2568</td>
</tr>
<tr>
<td>15</td>
<td>$1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 8 \rightarrow 6 \rightarrow 12 \rightarrow 10$</td>
<td>6.1492</td>
<td>0.2568</td>
</tr>
</tbody>
</table>

In the table 5, it shows link 3 and link 7 are more important than link 11 and link 15. we make
an explanation like this: if it is crowded on some critical links (like link 3 and link 7), it could cause
rescuer forced to change original plan and select other paths; and once choosing other paths, rescuer
has to spend more time and more energy on trip. Therefore, link 3 or link 7 will have more negative
effect on emergency rescue if their congestion degree is high. In view of this, these links is more
critical for rescuer than others.

③ the long rescue distance: from the node 1 to the node 14
When emergency occurs in the node 14, the emergency rescue path is:
$1 \rightarrow 3 \rightarrow 4 \rightarrow 6 \rightarrow 12 \rightarrow 20 \rightarrow 13 \rightarrow 21 \rightarrow 14$

Its generalized total impedance is $E^*_G(T) = 7.4968 \text{ min}$. For this path’s links ( link 3, link 6, link 20 and link 21), similarly the results is shown as following table 6:

<table>
<thead>
<tr>
<th>Delete link Number $i$</th>
<th>New shortest path $L_{G,i}$</th>
<th>New generalized total impedance $E^*_{G,i}(T)/\text{min}$</th>
<th>Importance degree $\Delta E^*_i(T)/\text{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 10 \rightarrow 8 \rightarrow 14 \rightarrow 14$</td>
<td>10.9781</td>
<td>3.4813</td>
</tr>
<tr>
<td>6</td>
<td>$1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 8 \rightarrow 6 \rightarrow 9 \rightarrow 7 \rightarrow 10 \rightarrow 8 \rightarrow 14 \rightarrow 14$</td>
<td>10.4875</td>
<td>2.9907</td>
</tr>
<tr>
<td>20</td>
<td>$1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 8 \rightarrow 6 \rightarrow 9 \rightarrow 7 \rightarrow 10 \rightarrow 8 \rightarrow 14 \rightarrow 14$</td>
<td>10.4875</td>
<td>2.9907</td>
</tr>
<tr>
<td>21</td>
<td>$1 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 8 \rightarrow 6 \rightarrow 9 \rightarrow 7 \rightarrow 10 \rightarrow 8 \rightarrow 14 \rightarrow 14$</td>
<td>10.4875</td>
<td>2.9907</td>
</tr>
</tbody>
</table>
In the table 6, we know that link 3 is the most critical link and the other 3 links (link 6, link 20, link 21) have the same importance degree \( \Delta E_i^*(T) \) values (2.9907 min). What’s more, there is an interesting phenomenon about link 3. Its importance degree \( \Delta E_i^*(T) \) value (3.4813 min) in the long rescue path is different with the value (1.4251 min) in the short or medium rescue path. It illustrates that link’s importance degree \( \Delta E_i^*(T) \) is changeable on different situations.

In order to explain the above phenomenon, this article tries to give a quantitative relationship among link’s generalized impedance \( E(T_i) \), travel time \( T_i \) and congestion degree \( x_i \). We draw a scatter diagram about these 3 parameters, and then adopt nonlinear interpolation method to fit a curve diagram as following:

![Scatter diagram and curve diagram](image)

Figure 6: Parameters relationship of each link in the GERT network

From the above figure, the result shows that: (1) there is an approximate curved surface relationship among these 3 parameters \( (E(T_i), T_i, x_i) \); (2) and because of nonlinear relationship, generalized impedance \( E(T_i) \) will change along with travel time \( T_i \) and congestion degree \( x_i \), which can lead importance degree \( \Delta E_i^*(T) \) to vary together; (3) compared with existing methods of critical link’s identification, this article reduces the method’s computational complexity from network level to path level and help rescuer find out right rescue path and critical link more accurately and timely. It is also satisfied with actual rescue requests.

5 CONCLUSION

Based on the emergency rescue, this article obtains relevant conclusions are as following:

(1) It is feasible to approximately estimate the rescue vehicle’s average travel time by fuzzy mathematics theory under poor information environment. The calculating results with two fuzzy operators are similar.

(2) In the paper, we establish the GERT network model and calculate link’s generalized impedance to reflect link selective probability.

(3) This article adopts traversing method to calculate each link’s importance degree on transportation network, which can effectively reduce previous method’s computational complexity and duly provide rescue path for emergency traffic management and control.

However, if considering that there are several rescue destination locations in emergency rescue, the paper’s method is no more applicable, and this is also the next study’s direction.
6 ACKNOWLEDGMENTS

This study is supported by the Specialized Research Fund for the Doctoral Program of Higher Education (no.20130184110020), Technological Research and Development Program of China Railway Corporation (no.2015G002-N and no.2014X006-A), Outstanding Innovation Talents Program of Southwest Jiaotong University (no. SWJTU-R-[2014]-1), as well as the Technological Research and Development Program of China Eryuan Engineering Group (no. KYY2015026).

REFERENCES


Berdica, K., An introduction to road vulnerability: What has been done, is done and should be done. Transport Policy, 9(2), 2002, pp.117-127.


Zhang, X., Day-to-day Road Network Vulnerability Identification Based on Network Efficiency. Journal of Transportation Systems Engineering and Information Technology, 17(2), 2007, pp. 176-182

2017, pp. 129-136