Research on the survivability of railway network based on complex network

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ABSTRACT

Research on the damage resistance of railway network based on complex networks is of great significance to ensure the normal operation of railway and enhance the destruction resistance of railway transport network. After analyzing the existing literature, a railway transport network was built based on complex network. Relative global efficiency is used as survivability indicator. Take the normal-speed railway network under the jurisdiction of the Chengdu Railway Administration as an example. Using MATLAB to simulate random attacks and deliberate attacks, it is found that the key sections and key nodes can greatly affect the survivability of network.

KEYWORDS: Survivability; Railway network; Complex networks; Random attack; Deliberate attack

1 INTRODUCTION

With the accelerating process of China's railway construction, China is the country which has the longest running railway line in the world, its railway networks become more complex as well. Based on the theory of complex networks, studying the survivability of railway network is of great significance to ensure the orderly operation of the railway. In recent years, domestic and foreign research on
survivability of railway network continues to deepen. Latora V and Marchiori M studied the topology characteristics of Boston metro transport network. Seaton K A and Hackett L M used similar method to analyze the topological characteristics of the metro networks in Boston and Vienna. Wang W etc. divided network survivability into robustness, cascading failure probability, etc. Kang put forward the definition of topological survivability. Yu etc. proposed the robust index from source node to target node for Adhoc network. Wang W etc. proposed the ruin-resistance property of a network is the ability to maintain its function when the nodes and edges of the network are damaged or destroyed. Therefore, the whole function of a network is closely related to the ruin-resistance property of nodes and sections. Based on the above research, this paper constructs the railway transportation network and establishes the index of the network survivability model, MATLAB is used to random attack and deliberate attack links and nodes of railway network which can simulate the links and nodes failure and propose the process of the survivability analysis of railway network. finally, a case is given to illustrate the effectiveness of the model and algorithm.

2 PROBLEM DESCRIPTION AND MODELING

The railway network is usually in normal condition. However, if some sections of the network are interrupted due to unexpected events, the performance of railway network will decline. The transportation capacity of railway network in case of emergency is defined as survivability of the railway network.

2.1 Network construction

The transport network is described abstractly as a weighted network connection diagram. 
\[ V = \{v_i | i = 1, 2, 3, \ldots, n \} \] is the transport network nodes set, \( E = \{e_k | e_k = (v_i, v_j) \} \) is the transport network sections set; \( A = (a_{ij})_{n \times n} \) is the adjacency matrix of transport network. If there is a direct section between node \( i \) and node \( j \), then \( a_{ij} = 1 \), otherwise \( a_{ij} = 0 \), \( n \) is the number of nodes in the network. \( D = (d_{ij})_{n \times n} \) is a distance matrix of transport network, \( d_{ij} \) is a direct section distance between node \( i \) and node \( j \), if \( a_{ij} = 1 \), \( d_{ij} = \infty \).

2.2 The indexes of Survivability of Railway Network

The relative global efficiency of the network can be used to show the overall connectivity of the network. In general, it is represented by \( E_{\text{glob}}(G) \), \( E_{\text{glob}}(G) \) is a number greater than or equal to 1. When \( E_{\text{glob}}(G) \) is equal to 1, it indicates that the network is not damaged. Using formula 1 and formula 2 to calculate the relative global efficiency of the network and reflect the survivability of railway network.

\[
E_{\text{glob}}(k) = \frac{1}{n(n-1)} \sum_{i\neq j} \frac{1}{d_{ij}^{\text{short}}}
\]  

(1)

In Formula 1, \( E_{\text{glob}}(k) \) is the global efficiency of the network after being attacked \( k \) times; \( d_{ij}^{\text{short}} \) is the shortest distance between node \( i \) and node \( j \).

\[
E_{\text{glob}}(G) = \frac{E_{\text{glob}}(k)}{E_{\text{glob}}(0)}
\]  

(2)

In Formula 2, \( E_{\text{glob}}(G) \) is relative global efficiency of the network, \( E_{\text{glob}}(0) \) is the global efficiency before network is damaged.
3 ANALYSIS PROCESS OF DESTRUCTION SURVIVABILITY

When the railway network suffers from a series of emergencies such as serious traffic accidents, natural disasters and terrorist attacks, some sections of the railway network are damaged and sections will be closed up. In computer simulation, random attack and deliberate attack are generally used to simulate damaged sections. Random attack refers to the random attack on the sections or nodes, deliberate attack is attacking the key nodes or key sections in the railway network. Using MATLAB to analyze the survivability of the railway network, and the process of the survivability analysis is as follows:

Step1: Construct the adjacency matrix and distance matrix of the railway network.
Step2: Use matrix $A$ and $D$ to calculate global efficiency $E_{glob}^{(0)}$.
Step3: Random attack or deliberate attack the section $A(i, j)$, let $A(i, j) = 0$ and $D(i, j) = \infty$ . If node $v_i$ is attacked, let all $i$ th row and $i$ th column of matrix $A$ be 0, and let all $i$ th row and $i$ th column of matrix $D$ be $\infty$ .
Step4: Continuous attacks return to Step3, otherwise proceed to the next step.
Step5: Calculate $E_{glob}^{(k)}$ and calculate the relative global efficiency.

4 CASE ANALYSIS ON SURVIVABILITY OF RAILWAY NETWORK

Taking the normal-speed railway of Chengdu Railway Administration as an example, we study its survivability in two cases of random attack and deliberate attack on sections and nodes. Chengdu Railway Bureau normal-speed railway operating lines and the distance between the main station are as shown in Figure 1.

![Figure 1. Chengdu Railway Bureau normal-speed railway operating lines](image)

and the distance between the main station
4.1 The Topology Characteristic of normal-speed Railway Network

The topology characteristics of the network reflect the survivability of the network to a certain extent, the topological characteristics mainly includes: The station degree distribution of the network, Cumulative degree distribution, Clustering coefficient, Betweenness.

The station degree distribution \( p(k) \) can reflect the most basic topology characteristics of the network, it indicate the probability that the degree of any point in the network is \( k \). The calculation formula of the cumulative degree distribution can be calculated by formula 3.

\[
p_k(k \leq K) = \sum_{k<k} p(k)
\]  

(3)

In Formula 3, \( p_k(k \leq K) \) means the probability that the degree is less than or equal to \( k \); \( k \) and \( K \) are both degree of stations.

According to the adjacency matrix \( A \), the degree distribution and the cumulative degree distribution of Chengdu railway Administration are shown in Figure 2.

![Figure2. Distribution of Station Degree and Cumulative Degree of Chengdu Railway Administration](image)

The average degree value of the Chengdu Railway Administration normal-railway network is 2.3, when the railway network is damaged, it is easy to cause the network to be unblocked. The highest degree points are Chongqing, Chengdu and Guizhou, and these stations are the key nodes in the railway network.

The clustering coefficient is used to describe the aggregation state of nodes in the network. The clustering coefficient of the network is calculated by using formula 4.

\[
C = \frac{\sum_{i=1}^{n} m_i}{\frac{1}{2} \sum_{i=1}^{n} k_i \cdot (k_i - 1)}
\]  

(4)

In Formula 4, \( k_i \) is the number of nodes connected to the node \( i \), \( m_i \) is the actual number of sections between the \( k_i \) nodes.

Using the adjacency matrix \( A \), calculated that the clustering coefficient of normal-speed railway in Chengdu Railway Administration is 0.103, the clustering coefficient of most sites are 0, which means the network has a low local connectivity.

The betweenness of the network is divided into the betweenness of the section and the betweenness of the node. The betweenness reflects the role and the importance of the corresponding node or section in the whole network which is of great practical significance. The betweenness of the section can be calculated by formula 5, and the betweenness of the node \( B_i \) can be calculated by formula 6.

\[
B(i, j) = \sum_{m,p \in \mathcal{P}, m \neq p} \frac{n_{mp}(ij)}{n_{mp}}
\]  

(5)
In Formula 5, $B(i, j)$ is the betweenness of the section $e(i, j)$, $n_{mp}$ is the number of shortest paths between nodes $m$ and $p$, $n_{mp}(ij)$ is the number of paths passing through the section $(i, j)$ which are shortest paths between nodes $m$ and $p$.

$$B_i = \sum_{j \in \mathcal{N}_i, k \neq i} \frac{d_{jk}(i)}{d_{jk}}$$ (6)

In Formula 6, $B_i$ is the betweenness of the node $i$, $d_{jk}$ is the number of shortest paths between nodes $j$ and $k$, $d_{jk}(i)$ is the number of paths passing through the node $i$ which are shortest paths between nodes $m$ and $p$.

Using the distance matrix $A$, we calculate the betweenness of the sections of the normal-railway network of Chengdu Railway Administration. The sections with larger betweenness are Chongqing to Suining, Chongqing to Ganshui, Guan’an to Sanhui town, and the nodes with large betweenness are Chengdu, Chongqing, Guiyang and Neijiang. These sections and nodes are responsible for a wide range of transportation and contact work, they play a vital role in the network.

![Figure 3. The relative global efficiency of the network in the case of attack sections and nodes](image)

From Figure 3, we can see that the relative global efficiency of attacking nodes is faster than that of attacking sections, and the relative global efficiency of network caused by deliberate attacks is faster than that of random attacks. When the 4 sections of network are deliberately attacked, the relative global efficiency is reduced to about 0.5, while the random attack of 4 sections leads to the relative global efficiency of the network to about 0.7. When we deliberately attack 2 nodes, the relative global efficiency of the network has dropped to about 0.4. However, when we randomly attack 5 nodes, the relative global efficiency of the network just drops to about 0.5. Therefore, in the daily operation of the railway and in the case of special circumstances, the key stations and key sections should be maintained first. Improving the survivability of critical sections and critical stations is of great significance to improve the survivability of the railway network.

5 CONCLUSION

In this paper, the analytical method of the survivability of railway network is proposed. Based on the construction of the railway transport network, MATLAB simulation is used to simulate random attacks and deliberate attacks on sections and nodes. The simulated attack results shows deliberate attacks will quickly lead to a decline in the overall function and connectivity of the network and the harm caused by attacking nodes is far greater than that by attacking sections. The protection of key sections and key nodes plays an important role in improving the survivability of railway network. The above analysis has certain significance to strengthen the resistance of railway network, and also has certain guiding significance to the railway protection.
REFERENCES


