Analysis of The Characteristics and Robustness of Metro Network Based On User Demand

Quan CHEN  
School of Highway, Chang’an University  
Middle-section of S. 2nd Rd, Xi'an, Shanxi,China  
917166531@qq.com

Chao-qun MA  
School of Highway, Chang’an University  
Middle-section of S. 2nd Rd, Xi'an, Shanxi,China  
19724207@qq.com

Shuang ZHANG  
School of Highway, Chang’an University  
Middle-section of S. 2nd Rd, Xi'an, Shanxi,China  
514439627@qq.com

ABSTRACT

Urban rail transit (URT) is an important support for the urban passenger transport system, it has gradually become the preferred means of transportation for residents, and plays a vital role in solving the problem of urban traffic congestion. More the emergencies tend to paralyze certain rail transit stations, thus affecting the overall efficiency of the rail transit network (RTN). So the analysis of the topological characteristics and robustness of urban traffic track networks is of great significance for optimizing the road network structure and ensuring stable operation. Based on the description of complex network theory, this paper uses Space L method to empirically analyze the indicators and distribution laws of urban rail transit networks (URTN) such as node degree, average path length and clustering coefficient in Shanghai, Chongqing and Xi'an, then the user demand factor is integrated into the robustness of each site to deliberate attacks to measure the node contributes to users with different transmission requirements. The results show that the URT network scale has a great impact on the urban network structure function. From the perspective of different users, the key nodes of the network may be completely different for the same metro network. And the research can provide a theoretical reference for the routine maintenance, safe operation and reasonable planning and design of the rail transit network.

KEYWORDS: Urban rail transit network, complex network, user demand, robustness

1 INTRODUCTION

The development of the subway is a sign of urban development. With the development of the city, the population is increasing, the public transportation network cannot solve the travel needs well. As public transportation, URT has the advantages of large volume, fast speed, safety and reliability, on-time and comfort, gradually become the preferred means of transportation for residents, and played a vital role in solving urban traffic congestion problems.
Complex network theory is to reveal the internal laws of complex systems, and to abstract the complex system using a graphical way (Li et al. 2015). Garrison and Marble began to use the graph theory knowledge to quantitatively analyze the topology of the transportation network and had defined the α, β, and γ indicators (Garrison and Marble 1964). Gattuso and Miriello (2005) first used graph theory to study the subway network structure of the subway, and gave weights to the network nodes to analyze the station's transfer ability and passenger flow attraction (Ye et al. 2017).

Liu and Song have analysed the characteristics and distribution rules of Guangzhou RTN, and focused on the overall network efficiency under the fault of the transfer station. Li Ying et al (2007) tested and evaluated the invulnerability of different network systems. The results show that the rail transit network has strong resistance to random failure of nodes and weak resistance to selective attacks. Vito (2002) has discussed the efficiency of Boston's metro network and mitigated the effects of deliberate attacks by identifying and protecting key sites.

The current research on rail transit networks is mostly based on the analysis of network characteristics and reliability, and there is still insufficient research on robustness (Han, Guo et al. 2012). As an effective supplement to the public transport network, the robustness is closely related to its ability to maintain efficient operation in the event of an emergency. However, in reality, different users often have different demands, people cannot accept higher consumption (such as transmission time or cost) due to their own limitations. In other words, the transmission through the node may be meaningless even if it is successful, so that the failure of the node may have a completely different impact on different users (JS, Mao et al. 2010).

Based on the description of complex network theory, this paper determines the quantitative evaluation of site and network robustness indicators, uses Space L method and Python software to empirically analyse the URTN of Shanghai, Chongqing and Xi'an such as node degrees, average path length, clustering coefficient and other indicators and distribution rules. Finally, the user demand factor is integrated into the robustness calculation of each site for deliberate attacks, to measure the contribution of nodes to users with different transmission requirements. It is expected to provide a theoretical reference for the daily maintenance, safe operation and reasonable planning and design of the rail transit network.

2 BACKGROUND

This paper uses three different network sizes of URT (Shanghai, Chongqing and Xi'an) to start model. Since then, the urban rail operation lines in Shanghai, Chongqing and Xi'an are 16 lines, 10 lines and 4 lines (including ring lines) respectively. Today, there are 350 stations in Shanghai, of which 56 are transfer stations, accounting for 16%, is the largest city in the country. Compared with Shanghai, the Chongqing URT has a total of 166 stations, including 16 transfer stations, the proportion is 9.6%, which belongs to the medium-scale network. In addition, the development of Xi'an RTN is relatively slow, which is only 89 stations and 6 transfer stations, accounting for 6.7%. The selected three URTN represent three different development levels of the network, as well as representative cities and network networks at different stages of development. It is feasible to use this as a basis for comparison.

3 METHODOLOGY
3.1 Line network structure analysis

The rich research experience in model research, evolution mechanism and structural stability is the reason why it is widely used in the field of complex network research. The network static geometry and its analysis method provided by graph theory and social network analysis are complex network research foundation (Wu and Di, 2004).

In order to reflect the travel characteristics of the URT static network structure, the Space L method is often used to construct the network structure of physical location, the structure established by the L method mainly reflects the geographical connection between stations in the rail transit network. Where the node represents the station and the side represents the line. When two adjacent stations belong to the same line, the two are connected by the side.

From the statistical characteristics of the topology of complex networks, the definition of key nodes is mainly reflected from the perspective of “significantly equivalent to criticality” which is means to quantitatively calculate the static analysis features such as degrees and betweenness, and sort and analyze them, and define the key nodes in the network through quantitative data.

1. Degree and degree distribution

For describing RTN, the node degree k refers to the number of other edges connected to a node directly. Degree distribution \( P(k) \) is the probability of nodes with a degree \( k \) over the whole nodes (Strogatz et al. 2001). It can be expressed as follows:

\[
P(k) = \sum k_i / N
\]

(1)

Where \( N \) is the total number of nodes in the network, \( k_i \) is the degree for each site.

The degree distribution of complex networks also has practical implications, In L space, the degree of nodes reflects the number of rail transit lines passing through each station. According to relevant statistics, the average node degree of the current rail transit network in most cities in China is between 2.0 and 2.5. The greater the average node degree, the higher the reachability of the network.

2. Clustering coefficient

The clustering coefficient of the network is the mean of the clustering coefficients of all nodes. In the L space, the clustering coefficient distribution reflects the intensity of the rail transit lines near each station. The larger the network clustering coefficient value, the higher the degree of aggregation of node distribution in the RTN, and the better the connectivity of the network. The calculation formula is:

\[
\overline{C} = \frac{1}{N} \sum_{i=1}^{N} C_i
\]

(2)

3. Average path length

Average path length is defined as the network and \( d_{ij} \) is the shortest path between any two nodes in the network. The average path length \( L \) is:

\[
L = \frac{2}{N(N-1)} \sum_{i \neq j} d_{ij}
\]

(3)

In the L space, the average shortest path length reflects the average number of stations between any two stations in the RTN system. The smaller the value of L, the smaller the average distance
between two sites, directly reflecting the reachability of the network.

4 Path indicator-node betweenness

Node betweenness counts the fraction of shortest paths passing through a given node and is an important evaluation index based on the path in the network.

\[ B_i = \sum_{s \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}} \]  

(4)

where \( \sigma_{st} \) is the number of shortest paths from node \( s \) to node \( t \), \( \sigma_{st}(i) \) is the number of shortest paths passing through node \( i \) between nodes \( s \) and \( t \). 

5 Network efficiency

Network efficiency is used to measure the quality of network connectivity. The better the connectivity of the network, the higher the network efficiency. Network efficiency is defined as the sum of the reciprocal of the weighted shortest path between all stations in the station to the network. The specific calculation is shown in Equation 5:

\[ E = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}} \]  

(5)

3.2 Network robustness based on user demand

Robustness is a basic attribute of a complex system. When the internal structure of the system and the external environment change, the evaluation system can maintain its system function. In real life, the research on the robustness of complex networks is of great significance. In the subway network, there are many uncertain factors, including terrorist attacks, overhauls, and emergencies. Under the influence of these uncertain factors, robustness becomes the decisive factor to ensure the normal function of the network.

Previous studies have used network efficiency changes to assess the robustness and vulnerability of the network, as shown in Equation 6:

\[ V_i = \frac{E_i - E}{E} \]  

(6)

where \( E_i \) indicates the network efficiency after the node \( i \) is removed from the network, and \( E \) is the network efficiency without removing the node \( i \).

However, this evaluation standard does not consider the acceptance level of passengers. Therefore, this section evaluates the robustness of urban rail transit networks (nodes and networks) according to the different needs of passengers, constructing the \( R \) user effective path subgraph and \( OD \) loss rate. It is recorded as the maximum tolerance factor of the user demand, which is defined as the transmission process that can successfully transmit information from the node \( s \) to the node \( t \) and consume no more than \( R \) times the original consumption. If \( L \) is recorded as the shortest path transmission consumption value of the acyclic path between any two nodes in the network, when the transmission consumption is lower than \( R \times L \), we can assume that the user has no requirement or restriction on the transmission consumption, and the transmission is acceptable. The sub-picture that can be used for all paths with the required \( R \) user transmission is the \( R \)-user effective path sub-picture of the user, indicating that only the nodes in the user sub-picture are meaningful to the user.
First, we define the OD loss rate $S_i(R)$ caused by the inter-station interval fault as the ratio of the number of trips that the node cannot reach the destination to the total number of trips after the node $i$ is removed from the attack, and is used to evaluate the URT robustness. The specific formula is as follows:

$$ S_i(R) = \frac{G - G(i, R)}{G} \quad (7) $$

Where $G = N \times (N - 1)$, $N$ is the total number of nodes in the network.

So the demand-based network robustness is:

$$ S(R) = \frac{\sum_i S(i, R)}{N} \quad (8) $$

4 RESULTS AND DISCUSSION

4.1 Analysis of topological characteristics of urban rail transit

This paper conducts a complex network statistical analysis of metro networks in Shanghai, Chongqing, and Xi'an. The above three metro networks represent different levels of metro networks. The comparison of basic conditions and topological characteristics is shown in Table 1. It can be seen that the metro networks of these three cities have the following commonalities: the average degree is close to 2, the network topology has both scale-free and small-world features (Zhang et al. 2013).

The larger the network size, the closer the connections between the vertices of the network become, the more obvious the characteristics of the small world. At the same time, the aggregation coefficient is small, even 0, large in diameter (relative to network size), and tends to select short sides. The node degree distribution of the network shown in Figure 1 is the maximum at 2, and the Beijing and Shanghai metro networks will have another peak at 4 degrees, representing the transfer station. Figure 2 has analyzed the path length of the metro networks. It can be seen that the short side is mainly selected in the subway network.

<table>
<thead>
<tr>
<th>Cities</th>
<th>Nodes</th>
<th>Edge</th>
<th>Degree</th>
<th>Betweenness</th>
<th>Diameter</th>
<th>Average path length</th>
<th>Clustering coefficient</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shanghai</td>
<td>350</td>
<td>401</td>
<td>2.29</td>
<td>0.04</td>
<td>49</td>
<td>16.32</td>
<td>0.009</td>
<td>0.09</td>
</tr>
<tr>
<td>Chongqing</td>
<td>169</td>
<td>181</td>
<td>2.14</td>
<td>0.08</td>
<td>38</td>
<td>13.98</td>
<td>0.003</td>
<td>0.11</td>
</tr>
<tr>
<td>Xi’an</td>
<td>89</td>
<td>91</td>
<td>2.04</td>
<td>0.15</td>
<td>25</td>
<td>10.23</td>
<td>0</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Figure 1: City Metro Network Degree Map
4.2 Network robustness analysis

According to the formula (6) of 3.2, the $S_i(R)$ of Shanghai, Chongqing and Xi'an is calculated and shown in Figure 3, 4, 5 and Figure 6:
It can be seen from the results of Fig. 3, 4, 5 and Fig. 6 that the robustness distribution of URTN in Xi’an, Chongqing and Shanghai has great similarity. For smaller R values, network user robustness and network degree distribution are related, and for larger R values, network user robustness is only related to the network degree structure. When the $R$ is 2, the robust performance tends to be stable. That is, the calculation result is determined by the urban rail transit network structure, and the correlation with the user demand is small. At the same time, it can be seen from Fig. 3, 4, 5 that the robustness of each station in urban orbit is smaller at the transfer station, and the transfer station to the first and last stations of the line is gradually increasing. The larger the robustness, the higher the robustness. The stronger the system's ability to maintain its original functions, this is consistent with its operating rules. In addition, it can be seen from Figure 6 that the railway network in Xi'an has the lowest robustness, and the robustness of the Chongqing railway network is second. The robustness of the Shanghai railway network is significantly higher than that of the other two cities, which means the network. The higher the level, the higher the level of robustness.

User requirements have a critical impact on the ordering of network nodes. For different network users, the ordering of nodes in the same network may be completely different. Therefore, an attack on the network may have completely different impacts on different network users, even There may be no impact at all for the appropriate user. Therefore, we can reduce the damage caused by the target attack to the user transmission by appropriately adjusting the user's requirements within the scope allowed by the user.

5 CONCLUSIONS

URTN is a kind of complex network, which has various characteristics of complex networks. This paper makes a deep research and analysis on the statistical characteristics and robustness of three URTN. The results show that the statistical indicators and robustness of the three cities have the same
probability distribution and large differences in numerical values. As a typical case of a complex network, the rail transit network has obvious small world characteristics. The larger the network size, the closer the connections between the vertices of the network become, the more obvious the characteristics of the small world. The similar characteristics are derived from the similarity of the distribution of urban metro network structure, and the similarities and differences can be explained by the complexity of the RTN. In the various construction stages of urban rail transit construction, we can take certain measures to ensure that its robustness is at a high level. For the initial stage, the number of lines is small, and the total number of stations is also small, it is necessary to focus on constructing a certain number of transfer stations to ensure that the network forms more laps and provide alternative routes when the fault occurs.

Compared with the previous analysis of the robustness of the rail transit network topology, this paper considers the contribution of different user requirements to the system. In the complex network of metro, user requirements greatly influence the ordering of network nodes. Therefore, when the same metro network is viewed from different users, the key nodes of the network may be completely different. For example, those nodes that are destroyed due to a destination attack may not have a fatal impact on their transmission for a particular user. So we can reduce the fatality of the target attack by appropriately adjusting the user's demand within the scope of the user's permission, greatly reduce the travel failure rate, improve the network robustness, avoid the sudden increase of the rail passenger flow, and achieve the purpose of greatly protecting the transmission of user information, to better achieve the purpose of network functions.

This paper combines two basic elements of the subway network: network topology and network user requirements. By learning the structure of the network, the robust analysis based on user requirements provides possibilities for risk analysis and decision making. However, there is no further research and analysis on the dynamics of urban rail transit network. From the previous literature and research, there are still many attack strategies and topological parameters for evaluating network reliability and robustness. Learning performance changes will be an interesting job. Another possible extension is a comparative analysis of the topology and dynamics of the RTS network on weekends and weekdays. This study may yield some interesting results.

6 REFERENCES


